5.4 Multiple-Angle Identities

Double -Angle Identities

memorize: $\sin 2u = 2 \sin u \cos u$

memorize: $\cos 2u = \begin{cases} \cos^2 u - \sin^2 u \\ 2\cos^2 u - 1 \\ 1 - 2\sin^2 u \end{cases}$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$ (don't need to memorize)

$\sin 2u = \sin(u+u)$
$= \sin u \cos u + \cos u \sin u$
$= 2 \sin u \cos u$

$\cos 2u = \cos(u+u)$
$= \cos u \cos u - \sin u \sin u$
$= \cos^2 u - \sin^2 u$
$= 1 - \sin^2 u - \sin^2 u$
$= 1 - 2\sin^2 u$
$= \cos^2 u - (1 - \cos^2 u)$
$= \cos^2 u - 1 + \cos^2 u$
$= 2\cos^2 u - 1$

Power-Reducing Identities

memorize: $\sin^2 u = \frac{1 - \cos 2u}{2}$

memorize: $\cos^2 u = \frac{1 + \cos 2u}{2}$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$ (don't need to memorize)

Half-Angle Identities

$\sin 225° = \sin\frac{45}{2} = \sqrt{\frac{1-\cos 45}{2}} =$

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

pos or neg: depends on the quadrant in which $\frac{u}{2}$ terminates

$$\tan\frac{u}{2} = \begin{cases} \pm\sqrt{\dfrac{1-\cos u}{1+\cos u}} \\ \dfrac{1-\cos u}{\sin u} \\ \dfrac{\sin u}{1+\cos u} \end{cases}$$

(don't need to memorize)

Homework: 5.4A pg 475: 1-23 odd
5.4B pg 475: 25-35odd not 29, 43,45

1. $\cos 2u$

$\cos(u+u)$

$= \cos^2 u - \sin^2 u$

3. $\cos 2u = 1 - 2\sin^2 u$

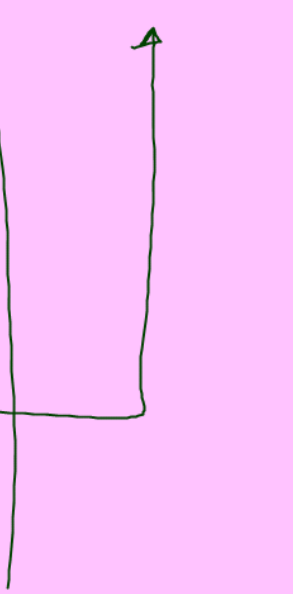
$\cos(u+u)$

$\cos u \cos u - \sin u \sin u$

$\cos^2 u - \sin^2 u$

$1 - \sin^2 u - \sin^2 u$

$1 - 2\sin^2 u$



$[0, 2\pi)$

5. $\sin 2x = 2\sin x$

$\sin 2x - 2\sin x = 0$

$2\sin x \cos x - 2\sin x = 0$

$2\sin x(\cos x - 1) = 0$

$\sin x = 0$ $\quad \cos x - 1 = 0$

$x = 0, \pi$ $\quad \cos x = 1$

$x = 0$

7. $\cos 2x = \sin x$

$\cos 2x - \sin x = 0$

$1 - 2\sin^2 x - \sin x = 0$

$-2\sin^2 x - \sin x + 1 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$w = \sin x$

$2w^2 + w - 1$

$(2w - 1)(w + 1)$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad x = \frac{3\pi}{2}$

$\sin 2x - \tan x = 0$

9. $2\sin x \cos x - \tan x = 0$

$\frac{2\sin x \cos x}{\cos x} - \frac{\sin x}{\cos x} = 0$ $\quad \cos x \neq 0$

$\frac{2\sin x \cos^2 x - \sin x}{\cos x} = 0$

$\sin x \left(\frac{2\cos^2 x - 1}{\cos x}\right) = 0$

$\sin x = 0$ or $\cos x \frac{2\cos^2 x - 1}{\cos x} = 0 \cdot \cos x$

$x = 0, \pi$

$2\cos^2 x - 1 = 0$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm \frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

13. $\sin 2\theta + \cos 3\theta$

$2\sin\theta\cos\theta + \cos(2\theta+\theta)$

$2\sin\theta\cos\theta + \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$2\sin\theta\cos\theta + (1-2\sin^2\theta)\cos\theta - 2\sin^2\theta\cos\theta$

$2\sin\theta\cos\theta + \cos\theta - 2\sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$2\sin\theta\cos\theta + \cos\theta - 4\sin^2\theta\cos\theta$

11. $\sin 2x + \cos x$

$2\sin x\cos x + \cos x$

$$\frac{1}{\sin x \cos x}$$

$$\csc x \cdot \frac{1}{\cos x} \frac{\sin x}{\sin x}$$

9. $\sin 2x - \tan x = 0$

$$2\sin x \cos x - \frac{\sin x}{\cos x} = 0$$

$$\sin x(2\cos x - \sec x) = 0$$

$\sin x = 0$

$x = 0, \pi$

$$2\cos x - \frac{1}{\cos x} = 0$$

$\cos x \square = \frac{1}{\cos x}$

$$\cos x\left(2 - \frac{1}{\cos^2 x}\right) = 0$$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$$2 = \frac{1}{\cos^2 x}$$

$2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

13. $\sin 2x + \cos 3x$

$2\sin x\cos x + \cos(2x+x)$

$2\sin x\cos x + \cos 2x\cos x - \sin 2x\sin x$

$2\sin x\cos x + (1-2\sin^2 x)\cos x - 2\sin x\cos x\sin x$

$2\sin x\cos x + \cos x - 2\sin^2 x\cos x - 2\sin^2 x\cos x$

$2\sin x\cos x + \cos x - 4\sin^2 x\cos x$

21. $\cos 4x = 1 - 8\sin^2 x\cos^2 x$

$\cos(2(2x))$

$w = 2x$

$\cos 2w$

$1 - 2\sin^2 w$

$1 - 2\sin^2(2x)$

$1 - 2\sin(2x)\sin(2x)$

$1 - 2\cdot 2\sin x\cos x \cdot 2\sin x\cos x$

$1 - 8\sin^2 x\cos^2 x$

17. $2\csc(2x) = \csc^2 x \tan x$

$2 \cdot \frac{1}{\sin 2x}$

$2 \cdot \frac{1}{2\sin x \cos x}$

$\frac{1}{\sin x \cos x}$

$\frac{\frac{\sin x}{\sin x}}{\frac{\sin x \cos x}{1}}$

$\frac{\sin x}{\sin x} \cdot \frac{1}{\sin x \cos x}$

$\frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x} = \csc^2 x \cdot \tan x$

$[0, 2\pi)$

7. $\cos 2x = \sin x$

$\cos 2x - \sin x = 0$

$1 - 2\sin^2 x - \sin x = 0$

$-2\sin^2 x - \sin x + 1 = 0$

$2\sin^2 x + \sin x - 1 = 0$

$w = \sin x$

$2w^2 + w - 1 = 0$

$(2w - 1)(w + 1) = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{3\pi}{2}$