

Homework: pg 469: 24,26,28,30

pg 475: 6,8,16,18,20,22,24,26,32

5.4B

43. $\cos^2 x$

$$= \sin^2\left(\frac{x}{2}\right)$$

$$= \left(\pm \sqrt{\frac{1 - \cos x}{2}}\right)^2$$

$$\cos^2 x - \frac{1 - \cos x}{2} = 0$$

$$2\cos^2 x - (1 - \cos x) = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$

$[0, 2\pi)$

$$45. \quad \tan\left(\frac{x}{2}\right) = \frac{1 - \cos x}{1 + \cos x}$$

$$\tan\left(\frac{x}{2}\right) = \tan^2\left(\frac{x}{2}\right)$$

$$0 = \tan^2\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right)$$

$$0 = \tan\left(\frac{x}{2}\right) \left(\tan\left(\frac{x}{2}\right) - 1 \right)$$

$$\tan\frac{x}{2} = 0 \quad \text{or} \quad \tan\left(\frac{x}{2}\right) - 1 = 0$$

$$\left(\tan\frac{x}{2}\right)^2 = \left(\frac{1 - \cos x}{1 + \cos x}\right)^2$$

$$\tan^2\frac{x}{2} = \frac{1 - \cos x}{1 + \cos x}$$

$$\frac{x}{2} = 0 \quad \text{or} \quad \frac{x}{2} = \pi$$

$$x = 0 \quad \text{or} \quad x = 2\pi$$

$$\tan\left(\frac{x}{2}\right) = 1$$

$$\frac{x}{2} = \frac{\pi}{4} \quad \text{or} \quad \frac{x}{2} = \frac{5\pi}{4}$$

$$x = \frac{\pi}{2} \quad \text{or} \quad x = \frac{5\pi}{2}$$

$$26. \quad \cos\left[\left(\frac{\pi}{2} - x\right) - y\right] = \sin(x+y)$$

$$\cos\left(\frac{\pi}{2} - x\right)\cos y + \sin\left(\frac{\pi}{2} - x\right)\sin y$$

$$\sin x \cos y + \cos x \sin y$$

$$\sin(x+y)$$

$$18. \quad 2 \cot 2x$$

$$\frac{2}{\tan 2x}$$

$$\frac{2}{\frac{2 \tan x}{1 - \tan^2 x}}$$

$$2 \cdot \frac{1 - \tan^2 x}{2 \tan x}$$

$$\frac{1}{\tan x} - \frac{\tan^2 x}{\tan x}$$

$$\cot x - \tan x$$

$$= \cot x - \tan x$$



$$20. \quad \sin 3x$$

$$\sin(2x + x)$$

$$\sin 2x \cos x + \cos 2x \sin x$$

$$2 \sin x \cos^2 x + (1 - 2 \sin^2 x) \sin x$$

$$2 \sin x \cos^2 x + \sin x - 2 \sin^3 x$$

$$\sin x (2 \cos^2 x + 1 - 2 \sin^2 x)$$

$$\sin x (2(1 - \sin^2 x) + 1 - 2 \sin^2 x)$$

$$\sin x (2 - 2 \sin^2 x + 1 - 2 \sin^2 x)$$

$$\sin x (3 - 4 \sin^2 x)$$

$$= (\sin x)(3 - 4 \sin^2 x)$$



$[0, 2\pi)$

$$28. \cos 2x + \cos 4x = 0$$

$$2\cos^2 x - 1 + \cos(2(2x)) = 0$$

$$w = 2x \\ 2\cos^2 w - 1$$

$$2\cos^2 x - 1 + 2\cos^2(2x) - 1 = 0 \checkmark$$

$$2\cos^2 x - 1 + 2(2\cos^2 x - 1)^2 - 1 = 0 \checkmark$$

$$2\cos^2 x + 2(4\cos^4 x - 4\cos^2 x + 1) - 2 = 0$$

$$2\cos^2 x + 8\cos^4 x - 8\cos^2 x + 2 - 2 = 0$$

$$8\cos^4 x - 6\cos^2 x = 0$$

$$2\cos^2 x(4\cos^2 x - 3) = 0$$

$$\cos^2 x = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

469

$$28. \cos\left(x - \frac{\pi}{4}\right)$$

$$= \frac{\sqrt{2}}{2}(\cos x + \sin x)$$

$$\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}$$

$$\cos x \left(\frac{1}{\sqrt{2}}\right) + \sin x \left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{1}{\sqrt{2}}(\cos x + \sin x)$$

$$\frac{\sqrt{2}}{2}(\cos x + \sin x)$$



475

16.

$$\cos(6x)$$

$$= 2\cos^2(3x) - 1$$

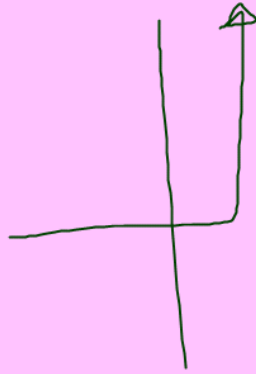
$$\cos(2(3x))$$

$$u=3x$$

$$\cos 2u$$

$$2\cos^2 u - 1$$

$$2\cos^2(3x) - 1$$



475

$$20. \sin 3x$$

$$= \sin x (3 - 4\sin^2 x)$$

$$\sin(2x+x)$$

$$\sin 2x \cos x + \cos 2x \sin x$$

$$2\sin x \cos^2 x + (2\cos^2 x - 1)\sin x$$

$$2\sin x \cos^2 x + 2\cos^2 x \sin x - \sin x$$

$$\sin x (2\cos^2 x + 2\cos^2 x - 1)$$

$$\sin x (4\cos^2 x - 1)$$

$$\sin x (4(1 - \sin^2 x) - 1)$$

$$\sin x (4 - 4\sin^2 x - 1)$$

$$\sin x (3 - 4\sin^2 x)$$



$$16. \quad \cos 6x = 2\cos^2(3x) - 1$$

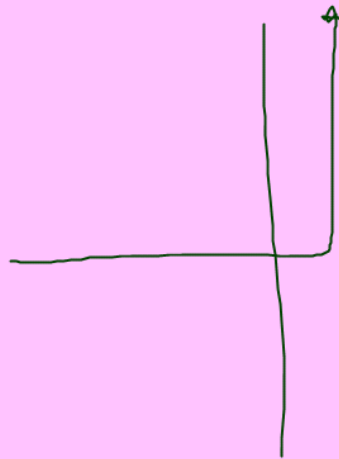
$$\cos(2(3x))$$

$$w = 3x$$

$$\cos 2w$$

$$2\cos^2 w - 1$$

$$2\cos^2(3x) - 1$$



$$22. \quad \sin 4x = (4\sin x \cos x)(2\cos^2 x - 1)$$

$$\sin(2(2x))$$

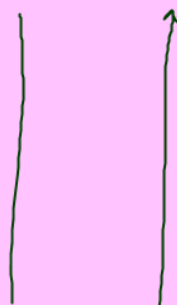
$$w = 2x$$

$$\sin 2w$$

$$2\sin w \cos w$$

$$2\sin 2x \cos 2x$$

$$2 \cdot 2\sin x \cos x (2\cos^2 x - 1)$$



$$18. \quad 2 \cot 2x$$

$$\frac{2}{\tan 2x}$$

$$\frac{2}{\frac{2 \tan x}{1 - \tan^2 x}}$$

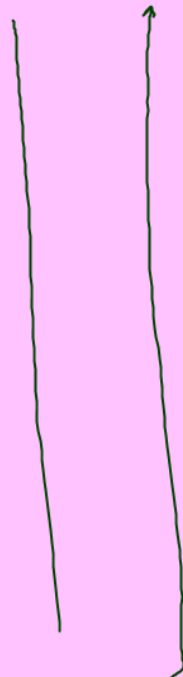
$$\cancel{2} \frac{1 - \tan^2 x}{\cancel{2} \tan x}$$

$$\frac{1 - \tan^2 x}{\tan x}$$

$$\frac{1}{\tan x} - \frac{\tan^2 x}{\tan x}$$

$$\cot x - \tan x$$

$$= \cot x - \tan x$$



$$2 \cot 2x$$

$$= \cot x - \tan x$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos 2x}{\sin x \cos x}$$

$$= \frac{\cos 2x}{\frac{1}{2} \sin 2x}$$

$$= 2 \cot 2x$$

$$24. \quad \cos 2x + \sin x = 0$$

$$1 - 2\sin^2 x + \sin x = 0$$

$$0 = 2\sin^2 x - \sin x - 1$$

$$w = \sin x$$

$$2w^2 - w - 1$$

$$(2w+1)(w-1)$$

$$0 = (2\sin x + 1)(\sin x - 1)$$

$$2\sin x + 1 = 0 \quad \text{or} \quad \sin x - 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$\sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{2}$$

S	A
T	C

475

$$32. \quad \tan 195^\circ = \tan \frac{390}{2} = \frac{1 - \cos u}{\sin u}$$

$$= \frac{1 - \cos 390}{\sin 390}$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \boxed{2 - \sqrt{3}}$$

$$195 = \frac{x}{2}$$

$$= x$$

$$\forall \sin x \cos^2 x = 0$$

$$\sin x = 0 \quad \vee \quad \cos^2 x = 0$$

$$x = 0, \pi$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$