5.3 Sum and Difference Identities

sin (u + v) = sin u cos v + cos u sin v

sin (u - v) = sin u cos v - cos u sin v

cos (u + v) = cos u cos v - sin u sin v

cos (u - v) = cos u cos v + sin u sin v

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

don't need to memorize

If one of the angles in a sum or difference is a quadrantal angle, then the sum-difference identities yield single-termed expressions called **reduction formulas**. For example:

$$\sin(x + \pi) = \sin x \cos\pi + \cos x \sin\pi$$

$$= \sin x(-1) + \cos x(0)$$

$$= \boxed{-\sin x}$$

Homework: 5.3A pg 468: 1-21 odd (not 7)

5.3B pg 468: 23-29odd, 35,37,42,47,49

1. $\sin 15° = \sin(45-30)$

$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{(\sqrt{3} - 1)}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

3. $\sin 75° = \sin(45° + 30°)$

$$= \sin 45° \cos 30° + \cos 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

5. $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}$$

$\frac{\pi}{3}-\frac{\pi}{4}$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$\frac{4\pi}{12}-\frac{3\pi}{12}=\frac{\pi}{12}$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

9. $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right) = \cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$$

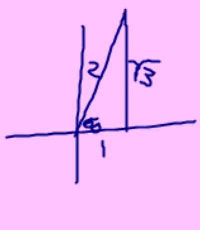
$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{2} - \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{3}}$$

$\frac{7\pi}{12} = \frac{3\pi}{12}+\frac{4\pi}{12}$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

$= \frac{\pi}{4}+\frac{\pi}{3}$



11. $\sin 42 \cos 17 - \cos 42 \sin 17^\circ = \sin(42-17) = \sin 25^\circ$

13. $\sin\frac{\pi}{5}\cos\frac{\pi}{2} + \cos\frac{\pi}{5}\sin\frac{\pi}{2} = \sin\left(\frac{\pi \cdot 2}{5 \cdot 2} + \frac{\pi \cdot 5}{2 \cdot 5}\right) = \sin\frac{7\pi}{10}$

15. $\dfrac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47} = \tan(19+47) = \tan 66$

17. $\cos\frac{\pi}{7}\cos x + \sin\frac{\pi}{7}\sin x = \cos\left(\frac{\pi}{7} - x\right)$

$= \cos\left[-\left(x - \frac{\pi}{7}\right)\right]$

$= \cos\left(x - \frac{\pi}{7}\right)$

11. $\sin 42^\circ \cos 17^\circ - \cos 42^\circ \sin 17^\circ = \sin(42^\circ - 17^\circ) = \boxed{\sin 25^\circ}$

15. $\dfrac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47} = \tan(19+47) = \tan 66$

23. $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$

$\sin\left(-\left(\frac{\pi}{2} - x\right)\right)$

$-\sin\left(\frac{\pi}{2} - x\right)$

$-\cos x$

$\sin x \underbrace{\cos \frac{\pi}{2}} - \cos x \underbrace{\sin \frac{\pi}{2}}$

$\sin x (0) - \cos x (1)$

$-\cos x$

29 $\tan\left(\theta+\frac{\pi}{4}\right) = \dfrac{1+\tan\theta}{1-\tan\theta}$

$$\frac{\tan\theta+\tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}}$$

$$\frac{\tan\theta+1}{1-\tan\theta}$$

37. $\sin\left(\frac{\pi}{2}-x\right) = \cos x$

$\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$

$\cos x - 0$

$\cos x$

49. $\cos 3x = \cos^3 x - 3\sin^2 x \cos x$

$\cos(2x+x)$

$\cos 2x \cos x - \sin 2x \sin x$

$\cos(x+x)\cos x - \sin(x+x)\sin x$

$(\cos x\cos x - \sin x \sin x)\cos x - (\sin x \cos x + \cos x \sin x)\sin x$

$(\cos^2 x - \sin^2 x)\cos x - (2\sin^2 x \cos x)$

$\cos^3 x - \sin^2 x\cos x - 2\sin^2 x \cos x$

$\cos^3 x - 3\sin^2 x \cos x$

23. $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$

$\underbrace{\sin x \cos\frac{\pi}{2}}_{0} - \cos x \underbrace{\sin\frac{\pi}{2}}_{1}$

$\sin x \cdot 0 - \cos x \cdot 1$

$-\cos x$

49. $\cos 3x$

$\cos(2x+x)$

$\cos 2x \cos x - \sin 2x \sin x$

$\cos(x+x)\cos x - \sin(x+x)\sin x$

$(\cos x\cos x - \sin x\sin x)\cos x - (\sin x\cos x + \cos x\sin x)\sin x$

$(\cos^2 x - \sin^2 x)\cos x - (2\sin x\cos x)\sin x$

$(\cos^3 x - \sin^2 x\cos x) - (2\sin^2 x\cos x)$

$\cos^3 x - 3\sin^2 x\cos x$

$= \cos^3 x - 3\sin^2 x\cos x$

$\boxed{\sin 2x = 2\sin x\cos x}$

$= \sin(x+x)$

$= \sin x\cos x + \cos x\sin x$

$= 2\sin x\cos x$