

### 5.3 Sum and Difference Identities

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

don't need to memorize

If one of the angles in a sum or difference is a quadrantal angle, then the sum-difference identities yield single-termed expressions called **reduction formulas**. For example:

$$\begin{aligned} \sin(x + \pi) &= \sin x \cos \pi + \cos x \sin \pi \\ &= \sin x (-1) + \cos x (0) \\ &= \boxed{-\sin x} \end{aligned}$$

Homework: 5.3A pg 468: 1-21 odd (not 7)

5.3B pg 468: 23-29 odd, 35, 37, 42, 47, 49

$$1. \sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{(\sqrt{3} - 1) \sqrt{2}}{2\sqrt{2} \sqrt{2}} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$$3. \sin 75^\circ = \sin(45^\circ + 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\begin{aligned}
 5. \quad \cos \frac{\pi}{12} &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\
 &= \cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\
 &= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{\pi}{3} - \frac{\pi}{4} &= \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{12} \\
 &= \frac{1 + \sqrt{3}}{2\sqrt{2}} \\
 &= \frac{\sqrt{2} + \sqrt{6}}{4}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) \\
 &= \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3} \\
 &= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} \\
 &= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \frac{7\pi}{12} &= \frac{3\pi}{12} + \frac{4\pi}{12} \\
 &= \frac{\pi}{4} + \frac{\pi}{3} \\
 &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}
 \end{aligned}$$



$$11. \sin 42^\circ \cos 17^\circ - \cos 42^\circ \sin 17^\circ = \sin(42^\circ - 17^\circ) = \sin 25^\circ$$

$$13. \sin \frac{\pi}{5} \cos \frac{\pi}{2} + \cos \frac{\pi}{5} \sin \frac{\pi}{2} = \sin \left( \frac{\pi}{5} + \frac{\pi}{2} \right) = \sin \frac{7\pi}{10}$$

$$15. \frac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47} = \tan(19+47) = \tan 66$$

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$$17. \cos \frac{\pi}{7} \cos x + \sin \frac{\pi}{7} \sin x = \cos \left( \frac{\pi}{7} - x \right) \\ = \cos \left[ - \left( x - \frac{\pi}{7} \right) \right] \\ = \cos \left( x - \frac{\pi}{7} \right)$$

$$11. \sin 42^\circ \cos 17^\circ - \cos 42^\circ \sin 17^\circ = \sin(42^\circ - 17^\circ) = \boxed{\sin 25^\circ}$$