2.3 Polynomial Functions of Higher Degree
Standard equation

$f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x^1 + a_0x^0$ $\quad a_n \neq 0$

Features: 1. continuous  **not** 

2. smooth, rounded turns,  not 

3. Leading Coefficient Rule:

n odd and a_n **>** 0	n odd and a_n **<** 0	n even and a_n **>** 0	n even and a_n **<** 0
			
$\lim_{x\to-\infty} x^{odd} = -\infty$	$\lim_{x\to-\infty} x^{odd} = \infty$	$\lim_{x\to-\infty} x^{even} = \infty$	$\lim_{x\to-\infty} x^{even} = -\infty$
$\lim_{x\to\infty} x^{odd} = \infty$	$\lim_{x\to\infty} x^{odd} = -\infty$	$\lim_{x\to\infty} x^{even} = \infty$	$\lim_{x\to\infty} x^{even} = -\infty$
$f(x) = 3x^5$	$f(x) = -3x^5$	$f(x) = 4x^4$	$f(x) = -3x^6$

A polynomial function of degree n has at most n - 1 local extrema (and turns) and at most n zeros.

Multiplicity:

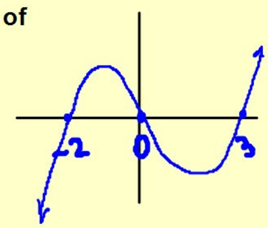
If f is a polynomial function and $(x - c)^m$ is a factor of f but $(x - c)^{m+1}$ is not, then c is a **zero of multiplicity m** of f.

$f(x) = x^1(x - 3)^1(x + 2)^1 = x^3 + \dots$ (looks like x^1, a line)

0 : mult 1 : crosses

3 : mult 1 : crosses (")

-2 : mult 1 : crosses (")

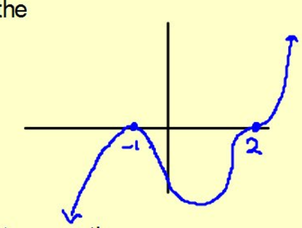


If a polynomial function f has a real zero c of **odd** multiplicity, then the graph of f **crosses** the x-axis at (c, 0) and the **value of f changes sign** at x = c.

$f(x) = (x - 2)^3(x + 1)^2 = x^5 + \dots$

-1 : mult 2 : kisses (looks like x^2, a parabola)

2 : mult 3 : crosses (looks like x^3, cubic)



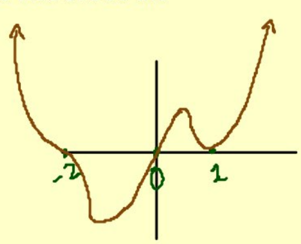
If a polynomial function f has a real zero c of **even** multiplicity, then the graph of f **does not cross** the x-axis at (c, 0)
(f **kisses** the x-axis at x = c) and the **value of f does not change sign** at x = c

$f(x) = x^1(x + 2)^3(x - 1)^2 = x^6 + \dots$

0, mult 1, cross, looks like a line

-2, mult 3, cross, looks like cubic

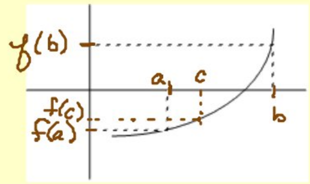
1, mult 2, kiss, looks like parabola

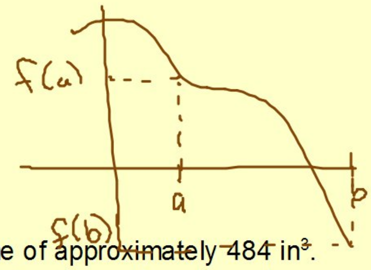


Intermediate Value Theorem

If a and b are real numbers with a < b and if f is continuous on the interval [a, b], then f takes on every value between f(a) and f(b). In other words, if y_0 is between f(a) and f(b), then y_0 = f(c) for some number c in [a, b].

In particular, if f(a) and f(b) have opposite signs (i.e., one is negative and the other is positive), then f(c) = 0 for some number c in [a, b].





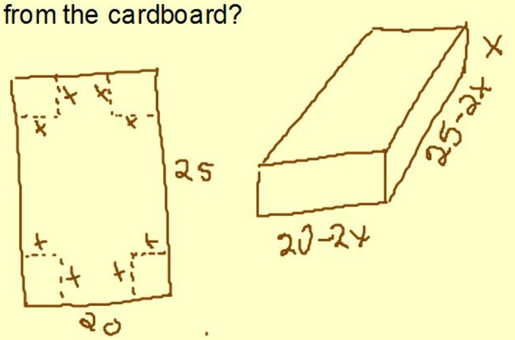
Ex. 9: DPC has contracted to make boxes with a volume of approximately 484 in³. Squares are to be cut from the corners of a 20-in. by 25-in. piece of cardboard, and the flaps folded up to make an open box. What size squares should be cut from the cardboard?

$V(x) = (20-2x)(25-2x)x = 4x^3 + \cdots$

$D: 0 < x < 10$

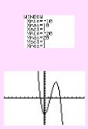
$x \approx 1.222$ in

$x \approx 6.871$ in



Homework: 2.3 pg 209: 9-12,18,19-27odd,37-43odd,49-53odd, 61,67,71-75

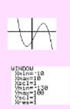
18.



$f(x) = +\sqrt{x}$

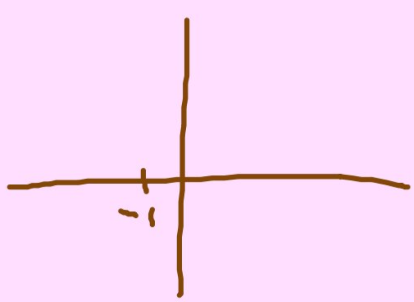
$f(4) = +\sqrt{4} = 2$

19.

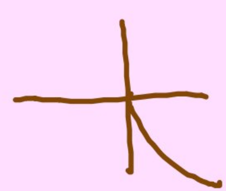


$x^2 = 9$

$x = \pm 3$



$f(x) = -\sqrt{x}$



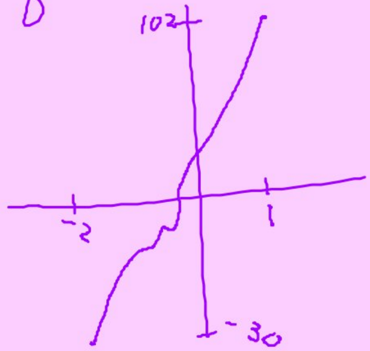
61. $f(x) = x^7 + x + 100$

$\lim\limits_{x \to -\infty} f(x) = -\infty$

$\lim\limits_{x \to \infty} f(x) = \infty$

$f(1) = +102$

$f(-2) = -30$



53. zeros: 3, -4, 6

$f(x) = (x-3)(x+4)(x-6)(-2)$

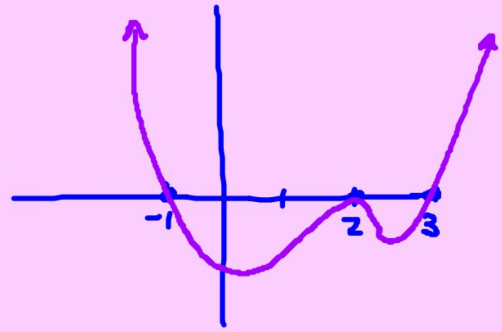
21. $f(x) = (x-2)^2(x+1)(x-3) = x^4 + \cdots$ 

Zeros:

-1, mult 1, cross (line)

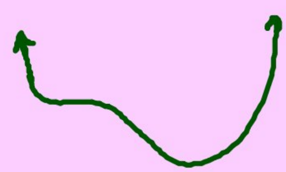
2, mult 2, kiss (parabola)

3, mult 1, cross (line)



23. $f(x) = 2x^4 - 5x^3 - 17x^2 + 14x + 41$

$\lim_{x \to -\infty} f(x) = \infty$



$\lim_{x \to \infty} f(x) = \infty$

2.2

51. $V = \frac{k}{P}$

$3.46 = \frac{k}{1.926}$

$V = kP$